Number Systems

Place value of numbers:

123 -> 1 hundred + 2 tens + 3 ones

In general a number ab is represented in terms of its place value as -> 10a+b

Classification of numbers:

Numbers are broadly classified as real numbers and complex numbers. We will be looking into only real numbers.

Real numbers can be classified as follows

- Rational Numbers- Numbers, which can be represented in the form p/q where q is not equal to zero
  eg: 2/3, 8/1

- Irrational- Numbers which cannot be represented in the form p/q where q not equal to zero
  eg: √3, √2

- Natural Numbers- > {1, 2, 3, …..∞} -> +ve counting numbers

- Whole numbers -> {0, 1, 2, 3 …..∞} – natural no: including zero

- Integers -> set of whole number along with -ve numbers {-∞.....-3, -2, ….0, 1, 2, ….∞}

- Even Numbers-> divisible by 2 {2, 4, 6…}
- odd Numbers-> not divisible by 2 {1, 3, 5, …}
- Prime Numbers-> no factors other than 1 & itself {2, 3, 5, 7, 11….}
- Composite Numbers -> numbers having other factors besides 1& itself {4, 6, 8, 9, 10, 12….}
- Coprimes - numbers which are primes with respect to each other. They have no common factors other than 1
  Eg:- 22,9. Co primes necessarily need not be prime numbers

Factors: To factor a number means to break it up into integers that can be multiplied together to get the original number
eg: 12 -> 2 x 6, 4 x 3 where 1, 2, 6, 4, 3, 12 are factors of 12
Note: For every number 1 and itself are always factors.

Finding factors of a number:

If a number is expressed in terms of its prime factors as $A^x \times B^y \times C^z$ then the number of factors of the given number is $(x + 1) \times (y + 1) \times (z + 1)$

The number of prime factors of this number is given by (x+y+z) and the number of unique prime factors of this number is no. of base values (a, b, c) (in above example it is 3 i.e. the three unique prime factors here are a, b and c)

Division Method: Dividend = (Divisor × Quotient) + Remainder

Divisibility by 2 - all even numbers

Divisibility by 3 - sum of digits of numbers is divisible by 3

Ex: 729 -> 7 + 2 + 9 = 18 is divisible by 3 so the number is divisible by 3

Divisibility by 4 – last 2 digits of numbers is divisible by 4

Ex: 724 - last 2 digits is 24 which is divisible by 4. Therefore, the number is divisible by 4

Divisibility by 5 – last digit 5/0

Divisibility by 6 – number should be divisible by 3 & 2 for it to be divisible by 6

Divisibility by 7 – multiply unit digit by 2, subtract that value from remaining digits. Keep continuing this process

Ex: 11347

Last digit of 11347 is 7 -> 7 x 2 = 14

1134 - 14 = 1120

last digit of 1120 is 0 -> 0 x 2 = 0

112 - 0 = 112

last digit of 112 is 2 -> 2 x 2 = 4

11 - 4 = 7 => div by 7

Divisibility by 8 - last 3 digits divisible by 8

2512 -> 512 divisible by 8 so the number is divisible by 8

Divisibility by 9 - sum of digits is divisible by 9

Ex: 729 -> 7 + 2 + 9 = 18 is divisible by 9. So the number is divisible by 9

Divisibility by 10 – last digit '0'

Divisibility by 11 -> find sum of alternatives digits & subtracts thus & equal to zero or multiple of 11.

Ex: 1 2 3 2 1

1 + 3 + 1 = 5

2 + 2 = 4

5 - 4 = 1 so not divisible by 11

Ex: 1 4 6 4 1

1 + 6 + 1 = 8

4 + 4 = 8

8 - 8 = 0 so divisible by 11

Divisibility by 12 - > Number should be divisible by 4 & 3

Divisibility by 13 -> Multiply last digit by 4. Add it to remaining digit. Keep continuing this process

Ex: 50661 -> 1 x 4 = 4

5066 + 4 = 5070 -> 0 x 4 = 0

507 + 0 = 507 -> 7 x 4 = 28

50 + 28 = 78.

78 is divisible by 13. So the number is divisible by 13

Divisibility by 17 -> multiply last digit with 5. Subtract the answer from the remaining digits. Keep continuing the process

Ex: 3978 -> 8 x 5 = 40

397 - 40 = 357 -> 7 x 5 = 35

35- 35 = 0

So divisible by 17.

Finding the unit digit of the given number with certain power value.

Pattern method:

1. If the unit digit of the given number is 0, 1, 5 or 6, then the same number will be the unit digit of the given number (i.e., 0, 1, 5 or 6).
2. If the unit digit of the given number is 4, then we have to check the power value ie., whether the given power is odd or even power. If it is odd power then, the unit digit is 4, if the power is even number, then the unit will be 6.
3. The same condition is applicable for 9. If it odd power then, the unit digit is 9, if it is even power, then the unit digit is 1.
4. If the unit digit of the given number is 2, 3, 7, or 8, then

Unit digit 2	$2^1 = 2$
	$2^2 = 4$
	$2^3 = 8$
	$2^4 = 6$
Unit digit 3	$3^1 = 3$
	$3^2 = 9$
	$3^3 = 7$
	$3^4 = 1$
Unit digit 7	$7^1 = 7$
	$7^2 = 9$
	$7^3 = 3$

	$7^4 = 1$
Unit digit 8	$8^1 = 8$
	$8^2 = 4$
	$8^3 = 2$
	$8^4 = 6$

Shortcut formulae:

1. A number being divided by d_1 and d_2 successively leaves remainders r_1 and r_2 respectively. Then remainder when the same number is divided by $d_1 \times d_2$

   Remainder = $d_1 \times r_2 + r_1$

LCM and HCF

1. LCM of fractions = LCM of numerator values / HCF of denominator values
2. HCF of fractions = HCF of numerator values / LCM of denominator values
3. HCF of two numbers × LCM of two numbers = Product of two number
4. Greatest number that will exactly divide x, y, z is = HCF of x, y, z.
5. Greatest number that will divide x, y, z and leaving remainders as a, b, c respectively is = HCF of ((x-a), (y-b), (z-c))
6. Greatest number that will divide x, y, z and leaving same remainder R in each case is = HCF of ((x-R), (y-R), (z-R))
7. Greatest number that will divide x, y, z and leaving same remainder in each case is = HCF of (|x-y|, |y-z|, |z-x|)
8. Least number that will exactly divisible by x, y, z is = LCM of x, y, z.
9. Least number that will divisible by x, y, z and leave the same remainder R in each case is = LCM of (x, y, z) + R
10. Least number that will divisible by x, y, z and leave remainders a, b, c respectively is = LCM of (x, y, z).- K [Where K=x-a=y-b=z-c]

Number Series

Questions on number series involve a series of numbers following a particular pattern

Questions asked from this topic are generally of two types

1) Find the missing number in the series
2) Find the wrong number in the given series

The common types of series that come include

I. Natural numbers

Eg: 1, 2, 3, 4, 5, 6

II. Even or odd series

Eg: 2, 4, 6, 8, 10

Eg: 1, 3, 5, 7, 9

III. Multiples of numbers

Eg: 4, 8, 12, 16, 20

IV. Arithmetic progression

Eg: 4, 7, 10, 13, 16

V. Geometric progression

Eg: 4, 8, 16, 32, 64

VI. Harmonic progression

Eg: 1/4, 1/7, 1/10, 1/13

VII. Square series

Eg: 1, 4, 9, 16, 25

VIII. Prime number series

Eg: 2, 3, 5, 7, 9

IX. Cube series

Eg: 1, 8, 27, 64

X. Difference based series

- constant difference: Eg: 3 7 11 15 19 23
- increasing/decreasing difference: Eg: 94 194 295 397
- Multilayer difference Eg: 37 41 49 61 77

Equations

Linear equation

Equations with highest power of variable as 1 are called as linear equations.

$ax + b = c$ / $ax + by = c$

$a \neq 0$

equation of form $ax^2 + bx + c = 0$ is called quadratic equation where $a \neq 0$

Linear equation has no solution if $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

if $a_1/a_2 \neq b_1/b_2$ & $c_1 = c_2 = 0$ then $x = y = 0$

$a_1/a_2 = b_1/b_2$ and $c_1, c_2 = 0$ then infinite no. of solutions

Quadratic equation

root → $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ b^2 - 4ac → discriminant

If roots of equation are and β then equation is $(x - \alpha)(x - \beta) = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

x^2 - (sum of roots) x + (product of roots) = 0

=> factorization /formula

e.g.: $y^2 + 11y + 10 = 0$

$= \frac{-11 \pm \sqrt{121-4\,x\,10}}{2} = \frac{-11 \pm \sqrt{81}}{2} = \frac{-11 \pm 9}{2} = -1, -10$

$(y - 4)(y + 5)$

factorization

$ax^2 + bx + c = 0$

ac => product

b => Sum

Find numbers satisfying the condition split middle term accordingly

Sign method – Quadratic equation

Sign table $Ax^2 + Bx + C = 0$

Bx	C	SR	BR
+	+	-	-
-	+	+	+
+	-	+	-
-	-	-	+

=> If B & C +Ve then both rots –ve

=> If B is –ve & C is +ve then both roots are +ve

=> If B +ve & C –ve then SR is +ve & bigger roots is –ve

=> If B is –ve & C is –ve then SR is –ve & BR is +ve

SR – Smaller root

BR – Bigger root

$x^2 – 7x + 10 = 0$

ac = 10

b = 7

We have to find two numbers whose product is 10 & sum is 7

Number = 5, 2

Roots are $\frac{5}{\text{coeff of A}}, \frac{2}{\text{coeff of A}}$ = 5/1, 2/1 = 5, 2

Using table sign of roots => +5, +2

Averages

Average = Sum of all values/No of values

Average is the equal distribution of total amount among given values.

Eg: In class average marks of 5 students is 60 means when we distribute the total marks equally among the 5 students each student gets 60 marks

Actually one student might get 0, other might get 100, 80 etc.,. While finding average we equally distribute the total marks among the students.

Weighted average:

If the average of n_1 quantities is x_1 and the average of n_2 quantities is x_2 then weighted average is $n_1x_1+n_2x_2/n_1+n_2$. This can be extended to any number of quantities.

Assumed mean/average:

As the name suggests assumed mean is an assumptions of the mean. It does not need to be correct/even close to actual mean. It is used to calculate actual mean.

Eg:- Consider numbers 40, 42, 48, 54 and we need to calculate average of these using assumed average method

Let's consider assumed average as 47

Deviation of each number from assumed average is

40 - 47 = -7, 42 - 47 = -5, 48 - 47 = 1, 54 - 47 = 7

Average of deviation = $\frac{-7-5+1+7}{4} = -1$

So actual average = 47 – 1 = 46

Include or Exclude:

In a group N values average is X, if K values included or excluded then their average also increase or decrease by a.

Then K values average = $\frac{(\text{Sum of (N+K) Values} - \text{Sum of N values})}{K} \Rightarrow \frac{[(N+K)(X+A)-(N)(X)]}{k}$

Replacement Case:

New Value - Replaced Value = No. of values * (Change in Average)

Average of remaining group

If the average of a group of 'n' elements is a, and the average of a group of 'n', (n, <n) elements out of 'n' elements is as, then the average of the remaining (n-n_1) elements will be $\frac{na-n_1a_1}{n-n_1}$

Percentages

Terminology:

The term percent means parts per 100 or "for every hundred". Thus, when we say a man made a profit of 20 percent we mean to say that he gained ₹20 for every hundred rupees he invested in the business, i.e., 20/100 rupees for each Rupee.

The abbreviation of percent is p.c. and it is generally denoted by %.

1. **A Percentage can be expressed as a Fraction.**

   10% can be expressed as 10/100 or 1/10.

   To express a percentage as a fraction divide it by 100 ⇒ a% = $\frac{a}{100}$.

2. **To express a fraction as a percent multiply it by 100**

   ⇒ $\frac{2}{3} = \left[\frac{2}{3} \times 100\right]\% = 66.66\%$

3. **A Percentage can be expressed as a Decimal. To express percentage as a decimal we remove the symbol % and shift the decimal point by two places to the left. For example, 10% can be expressed as 0.1**

   Example. Express $6\frac{1}{2}\%$ as a decimal.

   Sol. $6\frac{1}{2}\%$ = 13/2% = 6.5% = = 0.065

4. **To express decimal as a percentage we shift the decimal point by two places to the right and write the number obtained with the symbol % or simply we multiply the decimal with 100**

Example: Express 0.7 as a percentage.

Sol: $0.7 = 0.7 \times 100\% = \frac{7}{10} \times 100 = 70\%$

Fraction to Percentage Conversion Values:

Fraction	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	1/11	1/12
Percentage	50	33.33	25	20	16.66	14.28	12.5	11.11	10	9.09	8.33

Basic Formulae:

1. If the present population of a city is P and rate of increase or decrease is r% then Population after n years is = $P(1 \pm \frac{r}{100})^n$
2. If the present population of a city is P and rate of increase is r% then Population before n years is = $P(1 \pm \frac{r}{100})^n$

Note:

a) +ve sign indicates Increase

b) -ve sign indicates Decrease

3. Total Polled Votes = Valid votes +Invalid Votes
4. Valid votes = Winner Votes + Looser votes
5. Majority Votes =Winner Votes - Looser votes
6. If X is R% of a given number N, then N = $\frac{x \times 100}{R}$
7. If the price of commodity increases by R % then reduction in consumption so as not change the expenditure is = $\frac{r}{100+r} \times 100$
8. If the price of commodity increases by R % then reduction in consumption so as not change the expenditure is = $\frac{r}{100-r} \times 100$
9. If a value changed (decrease/increase) successively by x% and y%, then net change in value = $[\pm x \pm y \pm \frac{xy}{100}]$

Note:

a) +ve sign indicates Increase

b) -ve sign indicates Decrease

10. If A's income is r% more than that of B, then B's income is $\frac{r}{100+r} \times 100$ less than that of A.
11. If A's income is r% less than that of B, then B's income is $\frac{r}{100-r} \times 100$ more than that of A
12. A person spends x% of his monthly income on item A and y% of the remaining on item B. If he saves ₹P. Then the monthly Income = $[\frac{p \times 100 \times 100}{(100-x) \times (100-y)}]$

Important Note:

Consumption Based Questions

For fixed total expenditure

Price goes up by …%	Consumption comes down by …%
$\frac{1}{6}$ = 16.66	$\frac{1}{7}$ = 14.28
$\frac{1}{5}$ = 20	$\frac{1}{6}$ = 16.66
$\frac{1}{4}$ = 25	$\frac{1}{5}$ = 20
$\frac{1}{3}$ = 33.33	$\frac{1}{4}$ = 25
$\frac{1}{2}$ = 50	$\frac{1}{3}$ = 33.33
$\frac{1}{1}$ = 100	$\frac{1}{2}$ = 50

For fixed total expenditure

Price come down by …%	Consumption goes up by …%
$\frac{1}{5}$ = 20	$\frac{1}{4}$ = 25
20	33.33
33.33	50
50	100
75	300

Concept of effective percentage:

The effective percentage is calculated as $a + b + \frac{ab}{100}$ where a and b are percentage changes in 2 variable.

If the change is negative/there is a decrease in value then put a negative symbol in front of the percentage in the formula

This concept is very useful in profit and loss, simple and compound interest etc. Concept of effective percentage can be extended to more than two variables too.

Profit and Loss

Terminology:

Cost Price: The amount for which the shopkeeper buys the article is known as the Cost Price. It is usually written as CP or C.

Selling Price: The amount for which he sells the article is known as the Selling Price. It is usually written as SP or S.

Profit: The excess of the selling price over the cost price of an article is called the profit or the gain.

Loss: Sometimes, the shopkeeper has to sell the article for a price which is less than its cost price. In this case, the excess of the cost price over the selling price is called the loss.
Note: Profit and loss are always calculated with CP, as the base.

Marked Price: This is denoted as MP and is the price at which the article is usually marked. It is also known as Labelled Price or Printed Price or Tag Price or List Price.

Discount: The waiver of cost from the Marked Price that the trader allows a customer is called Discount.
Note:
If no discount is offered on the marked price, then the selling price is equal to marked price.
If some discount is offered on the marked price, then the selling price is equal to marked price less discount.

Basic Formulae:

1. Gain or Profit = Selling Price - Cost Price = SP - CP
2. Loss = Cost Price - Selling Price = CP - SP
3. Discount = MP - SP
4. Gain% = $\frac{Gain}{CP} \times 100 = \frac{SP-CP}{CP} \times 100$
5. Loss % = $\frac{Loss}{CP} \times 100 = \frac{CP-SP}{CP} \times 100$
6. When SP and gain% are given, then $CP = SP \times \frac{100}{100+Profit}$
7. When CP and gain% are given, then $SP = CP \times \frac{100+Profit}{100}$
8. When SP and loss% are given, then $CP = SP \times \frac{100}{100-Loss}$
9. When CP and loss% are given, then $SP = CP \times \frac{100-Loss}{100}$
10. Discount % = $\frac{Discount}{MP} \times 100 = \frac{MP-SP}{MP} \times 100$
11. When MP and discount % are given, then $SP = MP \times \frac{100}{100-Discount\%}$

Important Points:

- SP = 115% CP => Profit = 15% (since SP is 15% more than CP)
- SP = 120% CP => Profit = 20%
- SP = 108% CP => Profit = 8%
- SP = 144.8% CP => Profit = 44.8% and so on.

Similarly,

- SP = 90% CP => Loss = 10% (Since SP is 10% less than CP)
- SP = 80% CP => Loss = 20%
- SP = 76% CP => Loss = 24% and so on.

Shortcut Formulae:

1. If a man buys **a** items for ₹**b** and sells **c** items for ₹**d**, then the

   Gain or loss % = $\frac{ad}{bc} \times 100$

   Note: a) In case of gain percent, the result will be positive.

   b) In case of loss percent, the result will be negative.

2. If the CP of x articles = SP of y articles, then

   Gain or loss % = $\frac{x-y}{y} \times 100\%$

   Note: a) If x > y, it is % gain.

   b) If x < y, it is % loss.

3. A sells an article to B at a gain or loss of x%, and B sells it to C at a gain or loss of y%. If C pays ₹z for it to B, Then CP for $A = Z \times (\frac{100}{100 \pm x} \times \frac{100}{100 \pm y})$

   Note: a) If x or y is negative it indicates a loss.

   b) If x or y is positive it indicates a gain.

4. If two items are **bought** each at same price , then sold one item at Profit of x% and the other at loss of x%, Then overall profit or loss% is **Zero** (No Profit & No Loss)

5. If two items are **sold**, each at rupees P, one at a gain of X %and other at a loss of X%, Then overall profit or loss% is always a **loss** as ⇒ Loss = $\frac{x^2}{100}$

6. A dishonest dealer professes to sell his goods at cost price, but he uses a weight of lesser weight. Then Gain% = $\frac{True\ Weight - False\ Weight}{False\ Weight} \times 100$

7. A dishonest shopkeeper sells an item at a profit of x% and uses a weight which is y% less.

   Profit% = $\frac{x\%+y\%\ less\ weight}{100-y\%less\ weight} \times 100$

   Ex: - A dishonest shopkeeper sells an item at a profit of 10% and uses a weight which is 10% less. Profit% = $\frac{(10+10)\%}{100\%-10\%} \times 100 = \frac{20}{90} \times 100 = 22.22\%$

   Ex: - A dishonest shopkeeper sells an item at a loss of 5% and uses a weight which is 24% less. Profit% = $\frac{(-5+24)\%}{100\%-24\%} \times 100 = \frac{19}{76} \times 100 = 25\%$

8. A person sells two articles at the same price, one at a profit of P% and another at a loss of L% Then, net Profit or Loss % = $\frac{100(P-L)-2\times P\times L}{(200+P-L)}$

9. A person sells two articles at the same price, one at a loss of L% and another at a loss of N% Then, net Profit or Loss % = $\frac{100(-L-N)+2\times L\times N}{(200-L-N)}$

10. If two successive discounts on an article at x% and y% respectively, then the

    a. Overall Discount = $\left[x+y-\frac{xy}{100}\right]\%$

    or

    b. $100-(\frac{100-x}{100} \times \frac{100-y}{100} \times 100)$ (This method applicable for two or more values)

Simple and Compound Interest

Terminology:

Principal: The amount, which initially borrowed from a money, lender/the-initial amount deposited in a bank is called as Principal and it is denoted as P.

Time: The duration for which money borrowed/the duration for which the money is invested is called as Time and denoted with T.

Rate of Interest: A rate which is charged or paid for the use of money. An interest rate is often expressed as an annual percentage of the principal.it is denoted as R% per annum

Amount: The Amount, which is paid by the borrower/the amount received at the end of an investment including Principal and interest.

Interest: Interest is just the additional amount to be paid on the sum of money loaned/ borrowed or the extra amount earned from an investment

Interest is calculated into two ways:

- **Simple Interest:** In SI, the interest is calculated every year on the initial principal. So at the end of every year the interest calculated will be fixed amount.

  Formulae: SI $= \frac{PTR}{100}$

  $A = P + SI = P(1 + \frac{TR}{100})$

- **Compound Interest**: In compound interest at the end of every year while calculating interest for the next years, the interest earned in previous year is also taken into consideration. In CI we receive interest on the interest received in previous years

Formulae:

A = $P(1 + \frac{R}{100})^n$, interest payable annually

A = $P(1 + \frac{R}{100})^{2n}$, interest payable half-yearly

A = $P(1 + \frac{R}{100})^{4n}$, interest payable quarterly

CI = A - P

Calculation of CI using effective percentage

CI can be calculated using concept of effective %

If rate = a% and P is principal and n = 2 years then CI is calculated as follows

Net % = $(a + a + \frac{a \times a}{100})$ = $2a + a^2/100$

CI = P x $(2a + a^2/100)$

eg: r = 10%, n = 2, p = 1000. Find the compound interest for the given period

Net% = $(10 + 10 + \frac{10 \times 10}{100})$ = 21%

CI = 1000 x 21/100 = Rs. 210

For 3 years if CI is to be calculated it can be done with effective of as follows

- First calculate effective % of 2 years of interest
- Then calculate effective of third year interest along with effective of % of first 2 years.

  Eg: 10% for 3 years at CI

  $(10 + 10 + \frac{10 \times 10}{100})$ = 21% (effective percentage for 1st 2 years)

  $(21 + 10 + \frac{21 \times 10}{100})$ = 33.1% (effective percentage at the end of third year)

Difference (D):

1. The difference between SI & CI on a certain sum for two years at R% pa is
   $100^2 \times D = Pr^2$
2. The difference between SI & CI on a certain sum for three years at R% pa is
   $100^3 \times D = Pr^2(300 + R)$

Installments:

Equal annual installment to pay the debt (Borrowed) amount at S.I:

If annual payment of ₹X discharges a debt of ₹A due in n years at r% pa rate of interest at S.I , Then Amount of each Installment (x) is $\Rightarrow X = \frac{100 \times Amount}{100n + \frac{n(n-1) \times r}{2}}$

Shortcut method: (100+(n-1) r)% * X + - - - - -+ (100+2r)% * X+(100+r)% * X + 100% * X = A

Equal annual installment to pay the debt (Borrowed) amount at C.I:

Let the value of each equal annual installment = **₹a.**

Rate of Interest = R% p.a.

Number of installments per year= n.

Number of years = T.

∴Total number of installments = n x T.

Borrowed amount = B. Then, $a\left[\frac{100}{100+R} + \left(\frac{100}{100+R}\right)^2 + \cdots \left(\frac{100}{100+R}\right)^{n \times T}\right] = B$

Shortcut-Formulae:

1. If a certain sum of money amounts to **₹A1** in **t** years at CI & the same sum amounts to **₹A2** in **t+1** years at CI. Then rate of Interest = $\frac{A2-A1}{A1} \times 100$

2. If a certain sum of money amounts to **₹A1** in **t** years at CI & the same sum amounts to **₹A2** in **t+1** years at CI. Then Principal = $[A1 \times \left(\frac{A1}{A2}\right)^n]$

3. A certain sum of money amounts to **n** times itself in **t** years at SI, Then R% = $\frac{100*(n-1)}{t}$

4. A sum of money amounts to double itself at C.I in "t" years. Then in how many years will it amounts to 32 times (2^n times) => Time = 32 =2^5 (5 intervals of time is required) = 2^{5*t}.
   So Time = (5*t) years

5. A sum of money amounts to triple itself at C.I in "t" years. Then in how many years will it amounts to 27 times (3^n times) => Time = 27 =3^3 (3 intervals of time is required) = 2^{3*t}.
   So Time = (3*t) years

Example: If ₹650 amount to ₹790 in 4 years, then what sum of money will it amount to in 7 years at the same rate of interest?

Short-cut:

You can see that ₹140 is earned in 4 years

=> ₹35 is earned in 1 year => ₹35 x 7 =245

=> ₹245 is earned in 7 years etc.

Example: Find the CI on Rs.28000 for 3 years at 5% pa?

Short-cut:

1st Year = 140

2nd Year = 140 + 7 (Interest on 1st Year interest) =147

3rd Year = 140 + 7 + 7.35 (Interest on 2nd Year interest) = 154.35

CI = 140 + 147 + 154.35 = 441.35

Ratio and Proportion

When two numbers are represented in the form of another, this is done by expressing one number as a fraction of another

a : b if there are two numbers ax, bx then ax/bx = a/b a : b

Comparison of two quantities by division

Relation of one quantity with another

Duplicate ratio:

Ratio of squares of two numbers is called duplicate ratio of the two numbers

E.g.: $3^2/4^2 = 9/16$ is duplicate ratio of 3/4

Triplicate ratio:

Ratio of cubes $3^3/4^3 = 27/64$ is triplicate ratio of ¾

Sub duplicate ratio of square root ¾ is sub duplicate ratio of 9/16

Proportion:

Equality of two ratios is called proportions

if a/b = c/d then a, b, c, d are in proportions we write this as

a : b : : c : d → a, d extremes

c, b. means

-> product of extremes = Product of means

a x d = b x c

Fourth Proportional:

If a : b : : c : x then x is called fourth proportional of a, b, c and $x = \frac{b \times c}{a}$

-> Mean proportional

If a : x : : x : b , x is called mean or second proportional of a, b

$x^2 = ab$, $x = \sqrt{ab}$

If a/b = c/d then

$\frac{a+b}{b} = \frac{c+d}{d}$ (componendo)

$\frac{a-b}{b} = \frac{c-d}{d}$ (dividendo)

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (componendo / dividend)

$\frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$

if a/b = c/d = e/f = -- = k

Then a/b = c/d = e/f - - = K = $\frac{a+c+e--}{b+d+f--}$

Alligation and Mixture

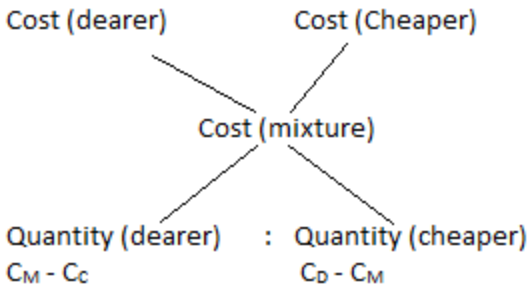
Mixtures – obtained by combining commodities of different type/different quantity.

eg: Rice of type 1 and type 2 are mixed, Water and alcohol is mixed

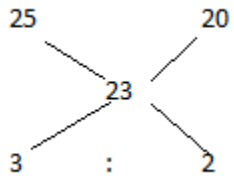
Alligation rule helps us to find the ratio in which the 2 commodities have to be mixed to obtain the mixture. This may be based on the price of the commodities/strength of the 2 commodities to form the mixture.

Alligation rule states quantities of the 2 commodities is inversely proportional to the difference of their costs with the mean cost (i.e., cost of the mixture). This helps us to find the ratio in which 2 or more ingredients at the given price / strength / value must be mixed to produce a mixture at a given price /value /strength

⇨ $\frac{Quantity\ (cheaper)}{Quantity\ (dealer)} = \frac{cost\ (dealer) - cost\ (mixture)}{cost\ (mixture) - cost\ (cheaper)}$



Eg: In what ratio should 1 mixture sugar costing 25 R/kg with sugar 2 costing 20 Rs/kg to get sugar costing 23 Rs/kg



If n vessels of equal size are filled with a mixture of P & Q in the ratio $p_1:q_1$, $p_2:q_2$ and content of all these vessels are mixed in one large vessel then

$$\frac{Quantity\ of\ P}{Quantity\ of\ Q} = \frac{\frac{p_1}{p_1+q_1} + \frac{p_2}{p_2+q_2} + \ldots + \frac{p_n}{p_n+q_n}}{\frac{q_1}{p_1+q_1} + \frac{q_2}{p_2+q_2} + \ldots + \frac{q_n}{p_n+q_n}}$$

Removal & Replacement

If a vessel has 'a' liters of milk from which 'b' liters is withdrawn and replaced with water. If this is done n times quantity of milk left after n operations is = $a(1-\frac{b}{a})^n$

Ages

Questions from topic ages is generally of the following types

1. Ages based on simple conditions
2. Ages based on complex conditions
3. Calculation of ages based on ratio
4. Calculation of ages based on average
5. Age difference based on ratio

Point to note

-> If current age is x, n years later age is (x+n)

-> If current age is x, n years before age is (x-n)

Partnership

Terminology:

Partnership: A partnership is a business with multiple owners, each of whom has invested in the business.

Partnership classified into mainly two ways:

Simple Partnership: All partners in the business do the investment for the same time period at the end of the business will share the profit based on ratio of their investments.

Compound Partnership: All partners in the business do the investment for different time periods at the end of a certain time period will share the profit based on the ratio of the product of their investments and time period.

Working Partner: Who invest in the business and work for the organisation.
Sleeping or Inactive Partner: Who invest in the business and does not work for the organization

Division of profits- Profit is divided among the partners based on the amount invested and the period for which the amount is invested.
Case 1:- Partners invest different amounts for same period

A invests x Rs. for P years
B invests y Rs. for P years

Ratio of share of profit between A & B is x:y

Ratio = A's Investment: B's Investment

Case 2:- Partners invest same amount for different periods

A invests x Rs. for P years

B invests x Rs. for q years

Ratio of share of profit between A & B is p:q

Ratio = A's Period of investment : B's period of investment

Case 3:- Partners invest different amount for different periods

A invests x Rs for p years

B invests y Rs for q years

Ratio of share of profit between A & B is xp : yq

Ratio = A's Investment * A's Time in partnership : B's Investment * B's Time in partnership

If investments of three persons are in the ratio is a : b : c and profits are in the ratio p : q : r , then the ratio of time = $\frac{p}{a} : \frac{q}{b} : \frac{r}{c}$

Time and Work

Terminology:

A person can do a piece of in "n" days , then 1 day work of A is = $\frac{1}{n}$

1-day work of a person is $\frac{1}{x}$, then no of days to finish the job is: x days.

The number of days required to finish work = $\frac{1}{\text{capacity}}$

Let us assume

p - No. of persons

H - No. of hours per day

D - No. of days

W - Work done

Then we can say

W α P (T = Constant) i.e. W$\uparrow$ => P$\uparrow$ & W$\downarrow$ => P$\downarrow$

W α T (P = Constant/) i.e. W$\uparrow$ => T$\uparrow$ & W$\downarrow$ => T$\downarrow$

W α H (P, D = Constant)

W α D (P, H = Constant)

W α PDH

$\frac{W_1}{w_2} = \frac{P_1 D_1 H_1}{P_2 D_2 H_2}$

Man day's concept /measurement of work

Work done can always be taken as no. of days x time for which they work

work = men x days

(or)

work = men x hours

If 12 men take 4 days to finish a work = 12 x 4 = 48 men days

If there are 6 m they will finish the work in 48/6 = 8 days

Formulas:

1. A can do a piece of work in x days and B can do the same in y days then both A&B together can do it in $\frac{xy}{x+y}$
2. If A, B, C can do a piece of work in x, y, z days respectively while working alone, they will together take $\frac{xyz}{xy+yz+zx}$ days to finish it.
3. A&B can do a piece of work in x days while A alone finish it in y days then B alone can do it in $\frac{xy}{y-x}$ days.
4. A & B can a piece of work in x days, B & C can do the same in y days while C & A can do it in z days. Then all of them (A, B & C) together take $\frac{2xyz}{xy+yz+zx}$days to finish it. .
5. If A is n times as fast as B, and is therefore able to finish a work in D days less than B. Then the time which they can do it working together in $\frac{D\times n}{n^2-1}$ days.
6. If X_1 men or Y_1 women can reap a field in D days, then X_2 and Y_2 women take to reap it in $\frac{(Days\times x_1\times y_1)}{x_2y_1+x_1y_2}$ days

Chain Rule:

If M_1 persons can do W_1 works in D_1 days in H_1 hours and M_2 persons can do W_2 works in D_2 days in H_2 hours

Then $\frac{M_1\times D_1\times H_1}{W_1} = \frac{M_2\times D_2\times H_2}{W_2} = M_1D_1H_1W_2 = M_2D_2H_2W_1$

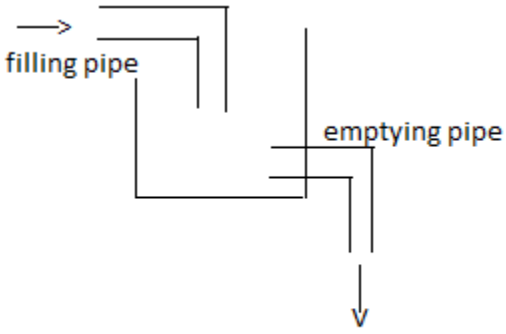
Pipes & cisterns:

Two types of pipes (1) filling pipe (2) emptying pipe/leak

Filling pipe does +ve work

Emptying pipe does –ve work

Assume a cistern / tank has an emptying pipe when can empty the tank in x hours and a filling pipe which can fill the tank in y hours



-ve sign represents leak/emptying pipes

Put together work done by both pipes in 1 hour is $(\frac{-1}{x} + \frac{1}{y})$

Time and Distance

Terminology:

Time: An amount of period is called as Time. Unit of time is hours or minutes or seconds.

Distance: The length between two points is called as Distance. Unit of distance is Km or metres.

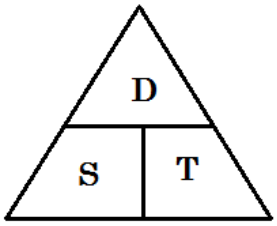
Speed: The distance covered in a particular time is called Speed. Unit of speed is kmph or mps.

Basic Formulae:

Distance = Speed × Time

Speed = Distance / Time

Time = Distance / Speed



Relation b/w D, T, S:

Distance Constant: Speed and Time are in inversely proportion

$$S \propto \frac{1}{T} \Rightarrow S * T = \text{Constant} \Rightarrow S_1T_1 = S_2T_2$$

Speed Constant: Distance and Time are in directly proportional.

$$D \propto T \Rightarrow \frac{D}{T} = \text{Constant} \Rightarrow \frac{D_1}{T_1} = \frac{D_2}{T_2}$$

Time Constant: Distance and Speed are in directly proportional.

$$D \propto S \Rightarrow \frac{D}{S} = \text{Constant} \Rightarrow \frac{D_1}{S_1} = \frac{D_2}{S_2}$$

Units Conversion:

Time:

1 min = 60 sec

1 hour = 60 min

1 hour = 3600 sec

Distance:

1 km = 1000 metres

1 metre = $\frac{1}{1000}$ km

Speed:

1 kmph = $\frac{5}{18}$ mps

1 mps = $\frac{18}{5}$ kmph

Average Speed:

The **average speed** of an object is the total distance traveled by the object divided by the elapsed time to cover that distance.

Average Speed = $\frac{\text{Total Distance travelled}}{\text{Total time taken to travel}}$

Relative Speed:

The speed of an object with respect to another object. When two objects are moving in the same direction, the relative speed is calculated as their difference. When the two objects are moving in opposite directions, the relative speed is computed by adding the two speeds

Relative Speed when both are in Same direction = $|S_1 - S_2|$

Relative Speed when both are in Opposite direction = $S_1 + S_2$

Important Points:

1. If two same intervals of distances covered in two different speeds x kmph, y kmph. Then the average speed = $\frac{2 \times x \times y}{x+y}$
2. If three same intervals of distances covered in three different speeds x kmph, y kmph, z kmph. Then the average speed = $\frac{3 \times x \times y \times z}{xy+yz+zx}$

Shortcut formulae:

1. A person travels a certain distance between two points. Having an average speed of x kmph, he is late by t_1 hours. However, with a speed of y kmph, he reaches his destination by t_2 hours earlier. Then Distance between two points is =

   $\frac{\text{Product of speeds}}{\text{Difference between speeds}} \times$ Difference between arrival times

   Distance = $\frac{x \times y}{|x - y|} \times (t_1 + t_2)$

2. Walking $\frac{a}{b}$ of his speed, a person is t_1 min late to his office.

   Then Usual time = $\frac{a \times change\ in\ time}{b - a}$

3. If two persons A and B start at the same time in opposite direction from two points and after passing each other, they complete journey in P and Q hours' resp.

   Then A's Speed : B's Speed = $\sqrt{b} : \sqrt{a}$

4. Without any stoppages if a bus travels a certain distance at an average speed of x kmph, and with stoppages it can covers the same distance at an average speed of y kmph. Then it stop time = $\frac{\text{Difference of speed}}{\text{Speed without stoppage}} \times 60$ min per hour

Problems on Boats & Streams:

Speed of Boat = u kmph

Speed of Stream = v kmph (It will also called as rate of current, rate of flow)

1. Downstream Speed = Speed of Boat + Speed of Stream = (U + V) kmph
2. Upstream Speed = Speed of Boat - Speed of Stream = (U - V) kmph
3. If x km is the rate of stream and a person takes n times as long to row up as to row down the river. Then speed of person in still water = $x \times \frac{n+1}{n-1}$

Calendar

Calendars topic mainly deals with finding out the day of the week on a given particular date. The concept odd days is of great importance to solve problems on calendars.

1. **Ordinary year**

A year having 365 days is called an ordinary year. February has only 28 days in ordinary years. 1 ordinary year = 365 days = 52 weeks + 1 day

2. **Leap year:**

A year having 366 days is called a leap years & February has 29 days in leap year. Generally every year divisible by 4 is called a leap year (except for centuries) 1 leap year = 366 days = 52 weeks + 2 days

3. **Odd days:**

The number of days more than the complete number of weeks in a given period is called odd days. An ordinary year has 365 days i.e. 52 weeks + 1 day. So the number of odd days = 1 similarly a leap year has 366 days i.e., 52 weeks + 2 days. So number of odd days = 2

4. **Counting of odd days:**

- Ordinary year has 1 odd day
- Leap years have 2 odd days
- A century has 76 ordinary years & 24 leap years

  100 years = 76 odd days + 24 x 2 odd days

  = 124 odd days = 17 weeks + 5 days

  so a century has 5 odd days
- 200 years has 10 odd days = 1 week + 3 days hence 3 odd days
- 300 years has 15 odd days = 2 week + 1 day hence 1 odd day

- 400 years has 20 + 1 odd day = 3 week, hence 0 odd day

The concept of odd days can be used to find which day of the week is a given date.

No: of odd days	Day
0	Sunday
1	Monday
2	Tuesday
3	Wednesday
4	Thursday
5	Friday
6	Saturday

Note: A century is a leap year if it is divisible by 400

Data Interpretation

Questions on data interpretation involve large amount of data given in the form of tables, pie charts, bar graph, line graph or some non-conventional format. Such questions deal with understanding, organizing and interpreting of the given data.

Form Representation of Data:

Data can be represented in multiple ways. The common representation of data include...

1) Bar graph
2) Tables
3) Line graph
4) Pie chart /circle graphs
5) Mixed graph

1) Bar graphs/histogram:

The information is given in terms of a bar graph where each bar represents some quantity / values in the given information.

2) Tables:

Tables is the most convenient way of providing data where the information is provided in terms of rows & columns of the given table.

3) Line graphs:

Line graph is a Cartesian graph or an x-y plot. Where the information is shown in terms of a line in x-y plot.

4) **Pie chart:**

A complete circle is divided into different sectors based on various percentages.

5) **Mixed graph:**

This is a combination of any of the above 4 graph.

Concepts required to solve questions on DI:

- Averages = Sum of all values / No. of values
- Ratio (a:b) = Comparison of like terms in its simplest form
- Percentage
  - I. x is what % of y = xy x 100
  - II. x is what % more (or) less than y = (x-y / y) x 100
  - III. Percentage change = $\left(\frac{(Final\ Value - Initial\ Value)}{Initial\ Value}\right)$ x 100

    $= \frac{Difference}{Initial\ Value}$ x 100

(Note:- % change can be % increase or % decrease)

Pie chart degree be % conversion

30 degree in pie chart represents

$\frac{30 \times 100}{360} = 8.33\%$

X degree represents $\frac{X \times 100}{360}\%$

Set Theory

A set is a collection of well-defined objects. Infinite Sets have fixed numbers of elements and infinite sets have infinite objects or elements some examples of sets are as follows

Set of natural numbers (N) = {1, 2, 3, 4, 5……}

Set of whole numbers (w) = {0, 1, 2, 3, 4...}

Set of integers (I) = { - - 2, -1, 0, 1, 2…}

Set of even numbers = {2, 4, 6, 8 …}

Set of odd numbers = {1, 3, 5, 7 …}

All above examples are infinite sets

Sets like {1, 2, 3, 4,} and {3, 6, 9, 12} are called infinite sets

Sets need not necessarily have numbers as elements in it

E.g.: set of Indians, Set of letters in alphabet

Operations on sets:

Let us consider 3 sets

A = { 1,2,3,4 } B = {4,5,6,7}

C= {7,8,9,10}

Union of sets:

Union of two sets A & B denoted by $A \cup B$ is the set of all elements either in A or in B or in both.

For above sets $A \cup B = \{1,2,3,4,5,6,7\}$

Intersection of Sets: Intersection of two sets A & B, denoted by $A \cap B$ is the set of all elements that belong to both A & B

E.g.: - $A \cap B = \{4\}$

Disjoint sets :

Two sets are said to be disjoint if $A \cap B \neq \phi$

E.g.: $A \cap C = \phi$ here A & C are disjoint sets

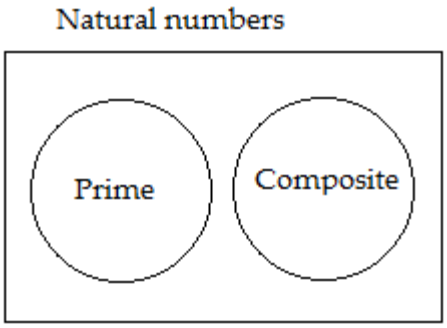
Difference of sets: Difference of two sets A and B, denoted by (A - B) is the set of elements that belong to A but not B

ex: $(A - B) = \{1,2,3\}$; $(B - A) = \{5,6,7\}$

$(B - C) = \{4,5,6\}$

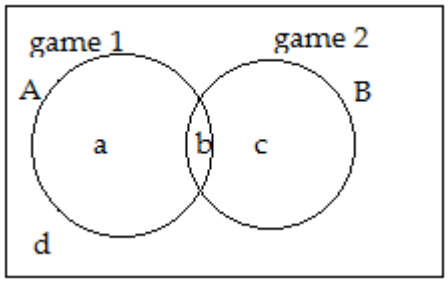
Venn diagram representation of sets:

Operations on sets can be represented as diagrams. A universal sets is represented by rectangle and a subset is represented by a circle in the rectangle. A venn diagram shows all the possible logical relationship between a infinite collection of sets:



Eg:

Students in a class



Total = A + B – (Both (A), none (AB)

Game 1 = a + b

game 2 = b + c

Total students = a + b + c + d

neither game 1 or nor game 2 = d

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Case lets

In this type information/date is given in the form of a paragraph instead of a table/graphs. Here the complete information in terms of facts & figures is represented in terms of sentences. We have to read this information & answer the questions followed.

In case lets raw information is given and this needs to be deciphered clearly to answer the questions many times writing down information in case lets in form of a table will helps us arrive we at answers faster.

Data Sufficiency

Questions on data sufficiency involves a question followed by 2 or 3 statements. We need to check the sufficiency of the statements to answer the question given i.e. we need to check if the data in the statements is sufficient to answer the question given. Many times, we need not find the actual solution of the question. Inferring whether the data in the statement would be sufficient in answering the question is enough.

Method of solving:

1. First check if the answer can be found using any one statement alone. Else combine the statements and check if the question can be answered.
2. Keep in mind that you are not required to actually solve the question. You just need to say if the statement is enough to answer the question. In the examination we shouldn't waste time solving the question
3. To solve questions on data sufficiency a clear understanding of concepts and problem-solving techniques of other topics in quantitative and reasoning ability is required.

Mensuration

Terminology:

Mensuration is a topic in Geometry, which is a branch of mathematics. Mensuration deals with length, area, and volume of different kinds of shape- both 2D and 3D. So moving ahead in the introduction to Mensuration, let us discuss what are 2D and 3D shapes and the difference between them.

What is a 2D Shape:

Moving ahead with our introduction to Mensuration let's discuss what is a 2D shape. A 2D shape is a shape that is bounded by three or more straight lines or a closed circular line in a plane. These shapes have no depth or height; they have two dimensions- length and breadth and are therefore called 2D figures or shapes. For 2D shapes, we measure area (A) and perimeter (P).

What is a 3D Shape?

The next step in introduction to Mensuration is finding out what is a 3D shape. A 3D shape is a shape that is bounded by a number of surfaces or planes. These are also referred to as solid shapes. These shapes have height or depth unlike 2D shapes; they have three dimensions- length, breadth and height/depth and are therefore called 3D figures. 3D shapes are actually made up of a number of 2D shapes. Also, know as solid shapes, for 3D shapes we measure Volume (V), Curved Surface Area (CSA), Lateral Surface Area (LSA) and Total Surface Area (TSA).

Important Terms:

Before we move ahead to the list of important mensuration formulas, we need to discuss some important terms that constitutes these mensuration formulas.

Area (A) – The surface occupied by a given closed shape is called its area. It is represented by the alphabet A and is measured in unit square- m^2/ cm^2.

Perimeter (P) – The length of the boundary of a figure is called its perimeter. In other words, it is the continuous line along the periphery of the closed figure. It is represented by the alphabet P and is measures in cm/ m.

Volume (V) – The space that is contained in a three-dimensional shape is called its volume. In other words, it is actually the space that is enclosed in a 3D figure. It is represented by the alphabet V and is measured in cm^3/ m^3.

Curved Surface Area (CSA) – In solid shapes where there is a curved surface, like a sphere or cylinder, the total area of these curved surfaces is the Curved Surface Area. The acronym for this is CSA and it is measured in m^2 or cm^2.

Total Surface Area (TSA) – The sum of the total area of all the surfaces in a closed shape is called its Total Surface Area. For example, in a cuboid when we add the area of all the six surfaces we get its Total Surface Area. The acronym for this is TSA and it is measured in m^2 or cm2.

Square Unit (m^2/cm^2) – One square unit is actually the area occupied by a square of side one unit. When we measure the area of any surface, we refer to this square of side one unit and how many such units can fit in the given figure. It is expressed as m^2 or cm^2, depending on the unit in which the area is being measure.

Cube Unit (m^3/ cm^3) – One cubic unit is the volume occupied by a cube of side one unit. When we measure the volume of any figure, we actually refer to this cube of side one unit and how many such unit cubes can fit in the given closed shape. It is written in m^3 or cm^3, depending on the unit that is being used to measure.

List of Basic formulae of 2D shape:

Square:

Area = a^2 (or) $\frac{d^2}{2}$ sq.units

Perimeter = 4a units

Diagonal d = $\sqrt{2}$ a units

Rectangle:

Area = l * b sq.units

Perimeter = 2(l+b) units

Diagonal $d^2 = l^2+b^2$ units

Scalene Triangle:

Area = $\sqrt{s \times (s-a) \times (s-b) \times (s-c)}$ sq. units $\left[s = \frac{(a+b+c)}{2}\right]$

Perimeter = (a + b + c) units

Equilateral Triangle:

Area = $\frac{\sqrt{3}}{4}a^2$ sq. units $\left[s = \frac{(a+b+c)}{2}\right]$

Perimeter = 3*a units

Height = $\frac{\sqrt{3}}{4}$(a) units

Isosceles Triangle:

Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$ sq units

Perimeter = 2a + b units ∴ (b = base length; a = equal side length)

Right-angled triangle:

Area = $(\frac{1}{2})b \times h$ sq. units

Perimeter = b + h + hypotenuse

Hypotenuse = $\sqrt{b^2 + h^2}$ units

Circle:

Area = πr^2 sq.units

Perimeter = $2\pi r$ units

Semi-Circle:

Area = $\frac{\pi r^2}{2}$sq. units

Perimeter (or) Circumference = πr +2r units

Sector:

Area = $(\frac{\theta}{360})$ πr^2 sq.units

Sector Length = $(\frac{\theta}{360})2\pi r$

Perimeter = 2 r + Sector Length

Parallelogram:

Area = (base * height) sq. units

Rhombus:

Area = $\frac{1}{2} \times d_1 \times d_2$ sq. units [d_1, d_2 are diagonals of rhombus]

Perimeter = 4 a units [a = side of rhombus]

Trapezium:

Area = $\frac{1}{2}$ × (Sum of parallel sides) × (height)

<u>List of Basic formulae of 3D shape</u>:

Cube:

Volume = $side^3$ cubic units

Surface Area = 6 × $side^2$ sq. units

Diagonal = √3 × (side) units

Cuboid:

Volume = (l × b × h) cubic units

Surface Area = 2(lb + bh + hl) sq. units

Diagonal = $\sqrt{(l^2 + b^2 + h^2)}$ units

Cylinder:

Volume = $\pi r^2 \times h$ cubic units

Curved Surface Area = $2\pi rh$ sq.units

Total Surface Area = $2\pi rh + 2\pi r^2 h = 2\pi r(r + h)$ sq. units

Cone:

Volume = $\frac{1}{3}\pi r^2 h$ cubic units

Curved Surface Area = $\pi \times r \times 1$ sq. units [l = Slope length]

Total Surface Area = $\pi \times r \times 1 + \pi r^2 = \pi r(r + 1)$ sq. units

Slope Length (l) = $\sqrt{(h^2 + r^2)}$ units

Sphere:

Volume = $\frac{4}{3}\pi r^2$ cubic units

Surface Area = $4\pi r^2$ sq. units

Prism:

Volume: (Area of base × height) cubic units

Permutation and Combination

Fundamental counting principles

1. **Principles of addition:** If one action can be done in 'm' different ways and another action can be done in 'n' different ways independent of the first one, then either of the two actions can be done in (m + n) ways.

   E.g.:- 5 different chocolates (c1, c2, c3, c4, c5)

   6 different fruits (F1, F2, F3, F4, F5)

   Any one of either, a chocolate or a fruit can be selected in 6+5 = 11 ways.

2. **Principle of multiplication**: If one action can be done in 'm' different ways and another action can be n different ways independent of the fruit one, then both the action together can be done in (m x n) ways.

   E.g.:- One can go from town A to B in 3 ways and town B to C in 2 ways. So total number of ways to go from A to C through B is = 3 x 2 = 6

3. **Fractional**

   Factorial n or n factorial is defined as product of first n natural numbers n! = 1 x 2 x 3 x ….n

   n! = (n-1)! X n

   0! = 1

4. **Permutations:**

   It's an arrangement which can be done using some or all of a given number of objects.

   E.g.:- Numbers of permutations of 3 objects a, b, c, taken 2 at a time are ab, bc, ca, ba, cb, ac

   Note: - Sequence | order is important

Permutation ➔ arrangement

Number of permutations of n different objects taken 'r' at a time is given as nPr = $\frac{n!}{(n-r)!}$

5. Combination

It is grouping / selection which can be done using some or all of the given objects.

E.g.: - Number of combinations of 3 objects a, b, c taken 2 at a time are ab, bc and ca.

Note: - sequence is not important.

Combination ➔ selection.

Number of combination of 'n' different objects taken 'r' at a time is given as nCr = $\frac{n!}{(n-r)!r!}$

➔ Number of permutations of 'n' objects taken all a time, where 'P' objects are similar of one type, 'q' objects are similar of second type and 'r' objects are similar of third type will be equal to $\frac{n!}{p!q!r!}$

6. Circular permutations

- The number of circular permutations 'n' different objects taken all at a time round a circle is (n-1)!
- The number of circular permutation of 'n' different objects taken all at a time round a circle (where a particular arrangement in clockwise direction is ……. Similar arrangement in anti-clockwise) is given as ½ (n-1)!

Probability

- **Random experiment**: An experiment whose outcome or result cannot be predicted even though all possible outcomes are known in advance is called a random experiment.
- **Sample space**: The set of all possible outcomes of an experiment is called sample space and is demand by 's'

  E.g.: - Throwing a dice

  S = {1, 2, 3, 4, 5, 6}

- **Probability**:

  Probability of an event's' is denote as P (E); O ≤ P (E) ≤ 1

  $$P(E) = \frac{No\ of\ favorable\ outcomes}{Total\ no\ of\ outcomes} = \frac{n(E)}{n(S)}$$

  $$P(E) = \frac{No\ of\ favorable\ outcomes}{Total\ no\ of\ outcomes} = \frac{n(S)-n(E)}{n(S)} = 1 - P(E)$$

  P(E) + P(E) = 1

  e.g.:- Assume we are tossing a coin.

  P (getting head) = ½

  P (getting head) = 1 – ½ = ½ = P (tail)

  odds in favour of an event 'E' = $\frac{P(E)}{P(E)}$

  = $\frac{n(E)}{n(S)-n(E)}$

  odds in against the event 'E' = $\frac{P(E)}{P(E)}$

  = $\frac{n(S)-n(E)}{n(S)}$

  e.g.:- If for an event odds in favour of the event are 3 : 4

then P(success of event) = $\frac{3}{3+4} = \frac{3}{7}$

Similarly if odds against even is given as 2 : 5 then P(failure of event) = $\frac{2}{2+5} = \frac{2}{7}$

Surds and Indices

- **Surds:** An irrational nth root of positive integer is called surd

  E.g.: - n√a = a^{1+n} E.g.: - √5, √2, 4√20

- **Index:** Power to which a number is raised

  E.g.: - a^n E.g.: - 2^5, 4^{12}, 3^5

- **Law of Indices**

  $a^m \times a^n = a^{m+n}$

  $a^m \div a^n = a^{m-n}$

  $(a^m)^n = a^{m \times n}$

  $a^m \times b^m = (a \times b)^m$

  $a^m \div b^m = (a \div b)^m$

  $a^0 = 1$

  $a^{1/m} = a^{-m}$

- **Law of surds**

  1. $\sqrt[n]{a} = a^{1/n}$
  2. $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$
  3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
  4. $(\sqrt[n]{a})^m = (a^{1/n})^m = a^{m/n}$
  5. $(\sqrt[n]{a})^n = (a^{1/n})^n = a$
  6. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

- **Rationalization** – Multiply numerator & denominator of a fraction with a common term so that surd in denominator is eliminated

  E.g.: - 3/√2

  $\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}}$

  E.g.: - $\frac{6}{\sqrt{5}+\sqrt{3}}$

  $= \frac{6}{\sqrt{5}+\sqrt{3}} \times \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})} = \frac{6(\sqrt{5}-\sqrt{3})}{(5-3)}$

  $= \frac{6(\sqrt{5}-\sqrt{3})}{2}$

  = 3(√5-√3)

For above question we have multiplied and divided the fraction by the conjugate of the denominator.

Algebra

Important Formulae

1. $(a+b)^2 = a^2 + 2ab + b^2$

2. $(a-b)^2 = a^2 - 2ab + b^2$

3. $(a+b)^2 = (a-b)^2 + 4ab$

4. $(a-b)^2 = (a+b)^2 - 4ab$

5. $(a+b)^3 = a^3+b^3+3ab(a+b) = a^3+b^3+3a^2b+3ab^2$

6. $(a-b)^3 = a^3- b^3-3ab(a-b) = a^3- b^3 - 3a^2b + 3ab^2$

7. $a^3+b^3 =(a+b)^3 - 3ab(a+b)$

8. $a^3- b^3 =(a-b)^3 + 3ab(a-b)$

9. $a^2 -b^2 = (a-b)(a+b)$

10. $a^3 +b^3 = (a+b)(a^2-ab+b^2)$

11. $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

12. $a^m \times a^n = a^{m+n}$

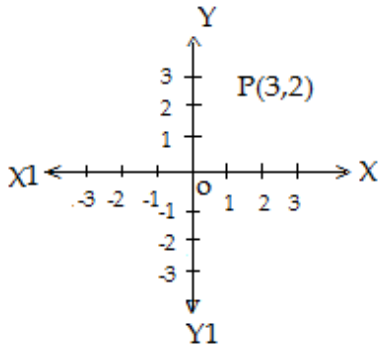
13. $a^m / a^n = a^{m-n}$

14. $(a/b)^{(m/n)} = (b/a)^{-(m/n)}$

15. $a^m / b^{-n} = a^m \times b^n$

Coordinate Geometry

Coordinate system: In Coordinate system a set of values or numbers are used to uniquely locate the position of a point in a given plane or a given space.



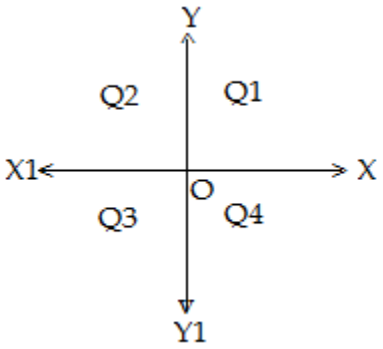
Cartesian system is the most commonly used coordinate system.

Here the plane is described with the help of two mutually perpendicular lines and any point in this plane can be uniquely located using two number values called coordinates and these values denote the distance of this point from the mutually 1r lines

Horizontal axis – x axis

Vertical axis – y axis

The x & y axis divides this plane into 4 quadrants. The values of the x and y coordinates in each quadrant is as below



- Q1 -> x -> +ve, y -> +ve
- Q2 -> x -> -ve, y -> +ve
- Q3 -> x -> -ve, y -> -ve
- Q4 -> x-> +ve, y -> -ve

Distance between two points

The distance between two points A (x_1, y_1) and B (x_2, y_2) in xy plane is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance of a point A (x, y) from the origin in x-y plane = $\sqrt{x^2 + y^2}$

The distance between the points A(x_1, y_1) and B(x_2, y) = $|x_2-x_1|$

The distance between points A(x, y_1) and B(x, y_2) = $|y_2- y_1|$

Section formula:

If any point p (x, y) divides the line segment joining the points A (x_1, y_1) and B(x_2, y_2) in the ratio m : n

1) Internally then

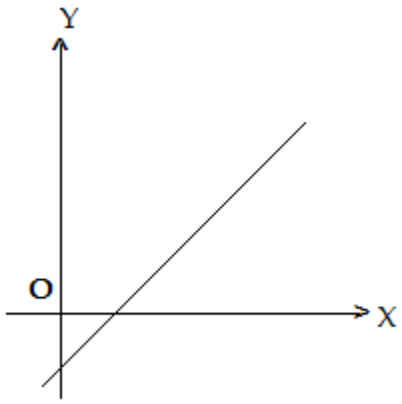
$P(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

2) Externally then

$P(x, y) = \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

Slope of straight line:

Slope also referred to as gradient represent the inclination of line w.r.t to the coordinate axis.



If 2 coordinates of line are (x_1, y_1) and (x_2, y_2) then

Slope = $\frac{y_2-y_1}{x_2-x_1}$ = tanθ

General form of a straight line

General form: ax+by + c = 0

Slope of the line = - $\left(\frac{coefficient\ of\ x}{coefficient\ of\ y}\right)$

= - a/b = m

- Equation of x axis: y = 0 => slope = 0
- Equation of y axis:- x = 0 => slope is undefined
- Line parallel to x axis:- y = k => slope = 0
- Line parallel to y axis:- x = k => slope = undefined

Equation of a straight line

Slope intercept form: - $y = mx+c$

Slope point form: - $(y-y1) = m(x-x1)$

Two point from: - $(y-y1) = \frac{y_2-y_1}{x_2-x_1}(x-x1)$

Intercept form: - $\frac{x}{a}+\frac{y}{b} = 1$

Parallel and perpendicular lines:

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

1) Coincident if $\frac{a_1}{a_2} = b_1/b_2 = c_1/c_2$
2) Parallel if slopes are equal

⇨ $- a_1/b_1 = - a_2/b_2 \Rightarrow a_1/a_2 = b_1/b_2$

3) Perpendicular if product of slope is -1

$(- a_1/b_1)(- a_2/b_2) = -1 \Rightarrow a_1a_2 + b_1b_2 = 0$

Distance of a point from a line

The length of a perpendicular from a point A(x1, y1) to a line ax+by + c = 0 is given by

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

Distance between two parallel lines

Let $ax+by+c_1 = 0$ and $ax+by+c_2 = 0$ be parallel lines, then distance between these lines is given by

$$\frac{|c1-c2|}{\sqrt{a^2+b^2}}$$

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are any three points then

Area of $\Delta ABC = \frac{1}{2} \{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}$

Points A, B, C are collinear if

1) Area of ΔABC = or
2) Slope of AB = slope of AC

- Centroid of $\Delta ABC = \left(\frac{x1+x2+x3}{3}, \frac{y1+y2+y3}{3}\right)$
- For circumcentre O(x,y), OA = OB = OC
- Incentre, $I(x, y) = \left(\frac{ax1+bx2+cx3}{a+b+c}, \frac{ay1+by2+cy3}{a+b+c}\right)$

  where a=BC, b = CA, C = AB

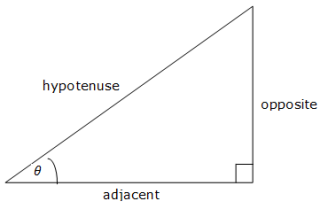
Trigonometry

Important Formulae

1. **Right Triangle Definition**

Assume that:

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$



$\sin\theta = \frac{opp}{hyp}$ $\quad$ $\text{cosec}\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\quad$ $\sec\theta = \frac{hyp}{adj}$

$\cos\theta = \frac{opp}{adj}$ $\quad$ $\cot\theta = \frac{adj}{opp}$

2. Tangent cotangent Identities

$\tan\theta = \frac{sin\theta}{cos\theta}$ $\quad$ $\cot\theta = \frac{cos\theta}{sin\theta}$

3. Reciprocal Identities

$\sin\theta = \frac{1}{cosec\theta}$ $\quad$ $\text{cosec}\theta = \frac{1}{sin\theta}$

$\cos\theta = \frac{1}{sec\theta}$ $\quad$ $\sec\theta = \frac{1}{cos\theta}$

$\tan\theta = \frac{1}{cot\theta}$ $\quad$ $\cot\theta = \frac{1}{tan\theta}$

4. Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

5. Even and Odd Formulas

$\sin(-\theta) = -\sin\theta$ $\quad \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\cos(-\theta) = \cos\theta$ $\quad \sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad \cot(-\theta) = -\cot\theta$

6. Double Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

7. Half Angle Formulas

$\sin\theta = \pm\sqrt{\frac{1-\cos(2\theta)}{2}}$

$\cos\theta = \pm\sqrt{\frac{1+\cos(2\theta)}{2}}$

$\tan\theta = \pm\sqrt{\frac{1-\cos(2\theta)}{1+\cos(2\theta)}}$

8. Sum and Difference Formulas

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

9. Product to Sum Formulas

$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$

$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$

10. Sum to Product Formulas

$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

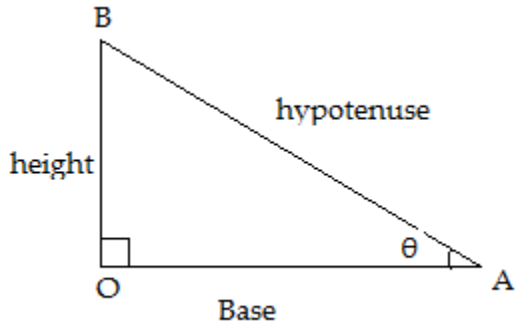
11. Co function Formulas

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad\quad$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad\quad$ $\sec\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec}\theta$

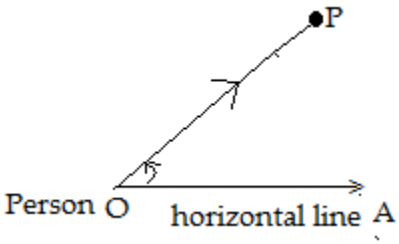
$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\quad\quad$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Height and Distance



Angle of elevation:

If a person looks at an object placed above the level of his eye, the angle which the line of sight makes with the horizontal is called the angle of elevation

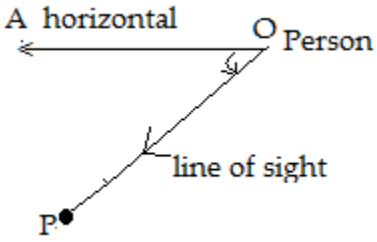


Eg:-

A person at O sees an object P angle of elevation of p from O = ∠AOP

Angle of depression

If a person looks at an object placed below the level of his eye, the angle which the line of sight makes with the horizontal is called angle of depression



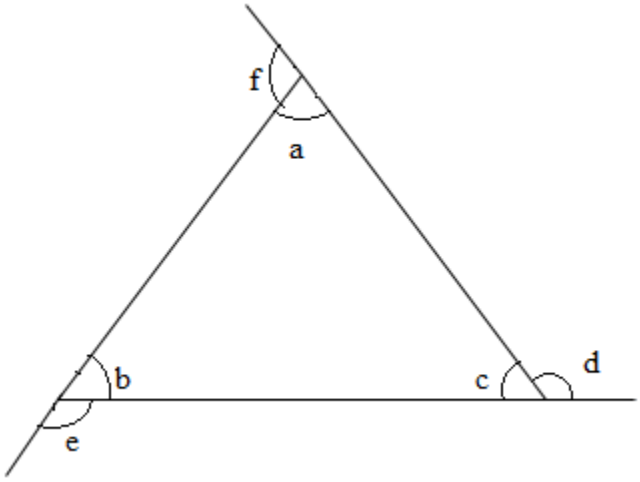
A person at O sees an object P below him

Angle of depression of P from O = ∠AOP θ

Geometry Triangle

TRIANGLES

- Any three-sided closed figure is called a triangle
- Sum of angles of a triangle is equal to 1800.
- The perpendicular drawn to aside from opposite vertex is called altitude to the side.
- The exterior angle of a triangle at each vertex equals to the sum of the two opposite interior angles.

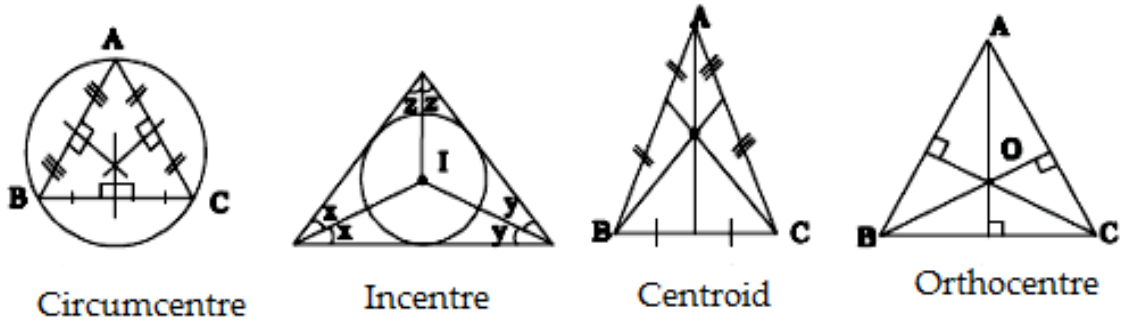


i.e., <d = <a + <b

<e = <a + <c

<f = <b + <c

- The perpendicular bisectors of the three sides of a triangle meet at a point called the circumcenter.
- The angular bisectors of the three angles inside a triangle meet at a point called incentre.
- The altitudes of the 3 sides of a triangle meet at a point called the orthocenter.
- The line joining the midpoint of a side with the opposite vertex is called the median drawn to the side and the median divides the triangle into two triangles of equal area.
- Three medians of a triangle meet at a point called centroid.



- Sum of any two sides of a triangle is always greater than the third side of the triangle. Similarly, the difference of any two sides of a triangle is less than the third side.
- If 1 represents the incentre of the given triangle ABC then
- $\angle BIC = 90^0 + \frac{1}{2}\angle A$

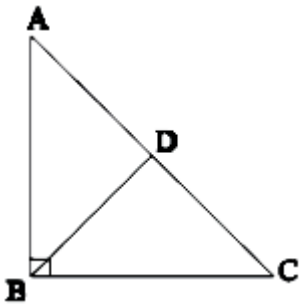
Classification of triangles:

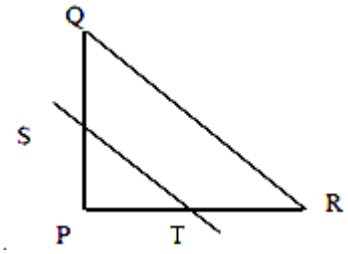
- A triangle with all sides equal & all sides equal to 600 is called an equilateral triangle.
- An isosceles triangle is one in which two sides are equal. Hence the two opposite angles are also equal.
- A scalene triangle is one in which all three sides and all three angles are different.

In the right-angle triangle, the length of the median drawn to the hypotenuse is equal to half the hypotenuse. The point D is also the circumcenter of the triangle.

i.e., BD = 1/2AC

And BD^2 = AD × CD





Note:

- A line drawn parallel to one side of a triangle, divides the other two sides in the same proportion.

  In this triangle if QR ||ST

  then PS/SQ = PT/TR = a/b

  Similarly, it can be stated that if ST || QR then length of ST = $\frac{a}{a+b}$ times length of QR.

Congruency & Similarity in triangles:

- Two triangles are said to be congruent any of the following conditions is satisfied
  - Three sides of one triangle is equal to the three sides of the other triangle (s –s – s rule)
  - Two sides and the included angle of the both the triangle are equal in measure (s – A – s)

- Two angles and one side of both the triangles are equal in measure (A –s – A)

➢ Two triangles are said to be similar triangles if

- Three angles of the both triangles are equal
- Two sides of both the triangles are in proportion to each other & included angle of the sides is same for both the triangles.
- Three sides of both the triangles are proportion to each other.

Note: Similar triangles are alike in shape only & corresponding angles of similar triangles are equal but corresponding sides are only proportional & not equal.

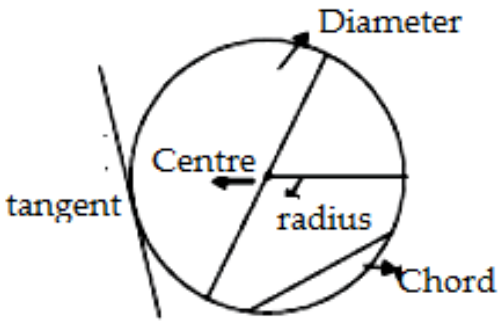
Geometry Circles

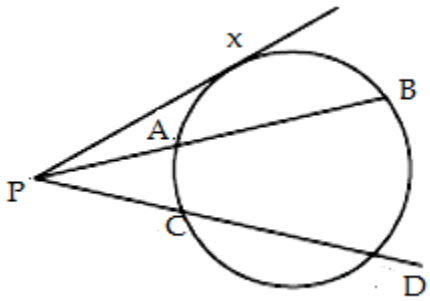
CIRCLES

A circle is a closed curve drawn in such a way that any point on the curve is equidistant from the centre of the curve (i.e. the centre of the circle)

Important notes:

- Distance from centre of circle, to any point on the circle is called radius of the circle.
- A line joining two points on the circumference of a circle is called chord.
- The largest chord of a circle is called the diameter of the circle and diameter passes through the centre of the circle dividing the circle into two equal halves.
- A line that touches the circle at only one point is called a tangent to the circle.

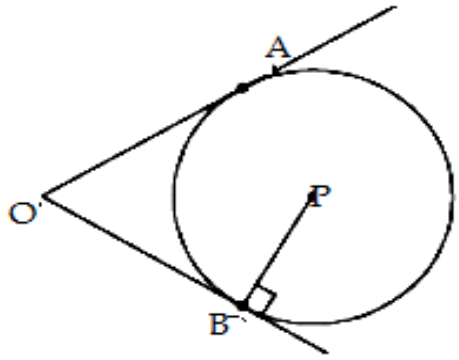




- A line intersecting the circle at two distinct point and extending outside the circle called a secant in above figure PAB and PCD are two secants.

  If PAB and PCD are two secants then PA.PB = PC.PD

  If PAB and PCD are secants and px is a tangent the circle then PA.PB = PC.PD = px2
- From any point outside a circle two tangents can be drawn to the circle and these two tangents will be equal in length.

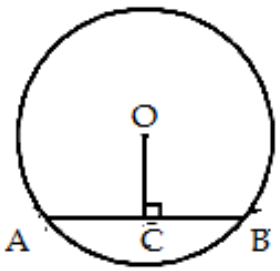


OA = OB

- A tangent is perpendicular to the radius drawn at the point of tangency & the converse is also true i.e., in above figure PB⊥r to tangent OB.
- A perpendicular drawn from the centre of a circle to a chord bisects the chord.

  Two chords equal in length will be equidistant from the centre

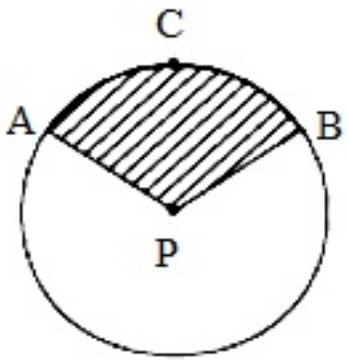
  Here AC = CB



Arcs and Sectors

A segment of a circle is called an arc and the area formed by an arc and the two radii at the end point of the arc is called a sector.

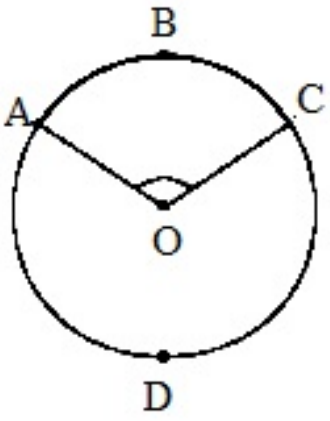
In this figure ACB is an arc and is represented as ACB.



The shaded region APB is called a sector

Arcs are of two types – minor arc and major arc generally when we refer to an arc, we refer to the minor arc.

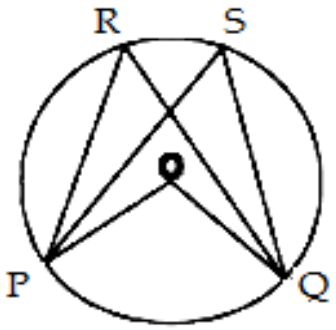
∠AOC is the angle subtended by the AC



Important notes:

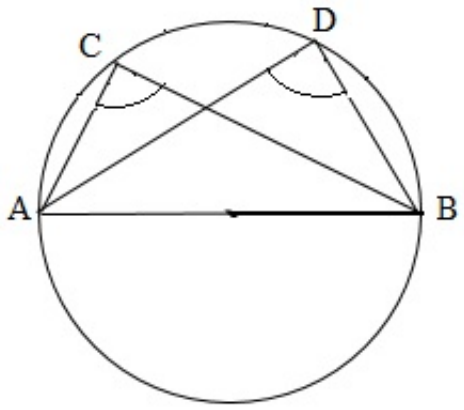
- The angle subtended by an arc at the centre of circle measures twice the angle subtended by the arc at any point on the circumference of the circle

  $\angle POQ = 2\ \angle PRQ = 2\ \angle PSQ$



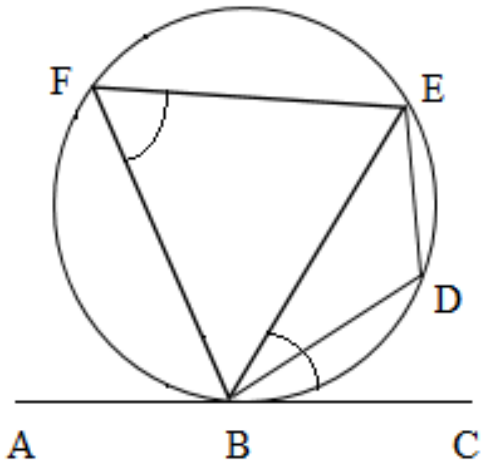
- The angle subtended in a semicircle a right angle

  $\angle ACB = \angle ABD = 90^0$



Alternate segment theorem:

This theorem states that the angle between a tangent and chord passing through the point of contact of the tangent is equal to the angle made by the chord in the alternate segment.



∠CBE = ∠BFE and ∠ABE = ∠BDE

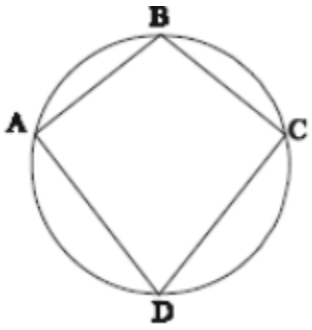
Note: Length of arc = $\frac{Angle\ of\ the\ arc}{360^0}$ x 2πr

Area of sector = $\frac{Angle\ of\ the\ sector}{360^0}$ x πr^2

Geometry Quadrilaterals

QUADRILATERALS

- Any four-sided closed figure is called a quadrilateral
- Sum of angles of a quadrilateral = 360 degrees
- The area of any quadrilateral can be calculated using formula. Area = ½ $d_1 d_2$ where $d_1$and d_2 are the diagonals of the quadrilateral.
- Rectangle, square, trapezium, rhombus, parallelogram are all different types of quadrilaterals.
- A quadrilateral inscribed in a circle is called as a cyclic quadrilateral. Opposite angles of a cyclic quadrilateral are supplementary.



$\angle B + \angle D = 180^0$

$\angle A + \angle C = 180^0$

POLYGON

Any closed figure with three or more than three sides is called a polygon. Polygons are generally divided into 2 types

- Convex polygon
- Concave polygon

A polygon with all interior angles less than 180^0 is a convex polygon.

A regular polygon is a convex polygon with all sides and all angles are equal

A polygon which has at least one interior angle measure greater than 180^0 is called a concave polygon.

Polygons are named based on number of sides they have as follows

No. of sides	Name of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Note:

- Sum of interior angles of a convex polygon is equal to (2n – 4) right angles. Here n represents the number of sides of the polygon
- In a regular polygon with n sides. Each interior angle of the polygon.
  I = $\frac{(2n-4)}{n}$ x 90^0 and each exterior angle = $360^0/n$
- A convex polygon with n sides has $\frac{n(n-3)}{2}$ diagonals. (A diagonal of a polygon is defined as any line joining any two non-adjacent vertices of a polygon).

Letter Series

Questions on letter series is based on letters in English alphabet. A series of letters is given and based on logic we have to find the missing letter or the wrong letter in the given series. Questions are generally based on position of letters in English alphabet position of A is 1, B is 2 ....Z is 26 and so on.

The different categories of questions that come from this section are

1) Alphabet series- Terms involve only letters of English alphabet
2) Alpha numeric series- Terms involve combination of numbers and letters
3) Letter pattern series- Terms involve letters from English alphabet. Here there are only few letters which keep repeating like a pattern. We need to identify the pattern